Basic Mathematics for Astronomy

A Manual for Brushing off the Rust

by

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Introduction

Astronomy is a fascinating science, from the distances to and inter-workings of stars and planets to the big questions like, "Where did it all come from?", "What is ultimately going to happen?" and "Are we alone?". Before we can begin to investigate these concepts we first have to master some basic mathematical skills that will allow us to talk about these astronomical concepts.

Most of these skills are grade school level arithmetic skills, so you don't have to be a mathematics major to understand the mathematical concepts necessary to do well in an introductory astronomy class. Honestly, you don't even have to like math. What you do have to do is be willing to spend the time necessary to master these skills. In some cases you may already have the mastery. In others, it may just be a matter of a little practice to "knock the rust off". Whatever the case, just like professors in other subjects can't take time from class to remind you how to read or write, we won't be spending any time in our astronomy class reviewing basic mathematical skills. Therefore this manual provides you a quick guide to the mathematical skills you will need and a basic set of practice exercises so you can check your level of mastery.

Each section in this manual begins with a description of each set of skills. You should read these descriptions and if you fully understand the concepts, try some of the sample problems. If you score well, move on to the next section. However, if you find that you do need to rethink these concepts, then study the description carefully and do all of the exercises section. The questions in the exercises section go more in depth than multiple choice exam questions will, but doing them will help you think through and master the concepts. Answers for all of the exercise questions are provided in the last section. Only use these answers after you are done with the exercise questions (looking at the answers ahead of time will not help you learn). Finally, the last part of each section is example exam questions. These are the types of questions you can expect to find on multiple choice exams related to these skills.

If at any point you are having difficulty with any of the sections, there are many further resources you can use. First, as you get to know people in the class you can form study groups. Study groups are excellent not only to work through this booklet, but to study for the class in general. Secondly, the web provides many sources of basic mathematical information. Try "googling" the term for whatever point you are stuck on. If this doesn't work, the University provides an entire academic unit, the Mathematics and Statistics Tutoring Center, which is there to help you with mathematical concepts. They are located at 208 Moseley Hall, (419) 372-8009, and provide services free of charge. Finally, your professor is there to help you as well. If it's a quick question ask before or after class, or send email. If you need to discuss a point in more depth, drop by office hours.

Remember mathematics, like all of the other skills you are gathering here at the University, is a tool you need to master to help you succeed. If your earlier mathematics training was lacking due to poor curricula or poor teachers, your college career is your last chance to master these very important skills.

Names of Numbers

Description

One of the first mathematical challenges we find ourselves facing in astronomy is dealing with very large numbers. Unless you are an accountant for the federal government, these are numbers you just don't encounter in everyday life. Before we can begin to even talk about them we need words to name these very large numbers.

Numbers are named following a very straightforward pattern. Starting with the familiar, one thousand, and progressing to larger numbers it goes as follows:[1]

1,000	one thousand
1,000,000	one million
1,000,000,000	one billion
1,000,000,000,000	one trillion

Notice how each set of three zeros (separated by a comma) has a new name. Note the order of the names, thousand, million, billion, trillion, each greater than the previous. The above are the names you need to be familiar with for you introductory astronomy class, but in case you are interested, the names continue.

1,000,000,000,000,000	one quadrillion
1,000,000,000,000,000,000	one quintillion
1,000,000,000,000,000,000,000	one sextillion
1,000,000,000,000,000,000,000,000	one septillion
1,000,000,000,000,000,000,000,000,000	one octillion
1,000,000,000,000,000,000,000,000,000,000	one nonillion
1,000,000,000,000,000,000,000,000,000,000,000	one decillion
...	etc.

Each name represents a number one thousand times larger than the previous, and since we can always multiply any number by 1,000 there is no largest number, and the naming convention continues. But concentrate on the first table, thousand, million, billion, and trillion. If you can keep these straight, which is larger than which, and how many digits each has, you are set for introductory astronomy.

Now of course there are numbers in between one million and one billion, one billion and one trillion and we need names for these as well. Fortunately we don't have to memorize billions of names since all of these numbers follow a very straight forward naming convention, and if you can count to one thousand, you already know it. Lets start with million.

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1 The British and some other Commonwealth countries use a different naming progression. The one presented here is the American system which is becoming the most commonly used, even in England.

1,000,000	one million
2,000,000	two million
3,000,000	three million
5,000,000	five million
10,000,000	ten million
20,000,000	twenty million
50,000,000	fifty million
100,000,000	one hundred million
250,000,000	two hundred fifty million
585,000,000	five hundred eighty-five million

Notice that the six zeros to the right are just there to tell us "million" and the digits left of these zeros are telling us how many million. This pattern repeats for billion, trillion, and all numbers larger. For example:

2,000,000,000	two billion
10,000,000,000	ten billion
13,000,000,000	thirteen billion
151,000,000,000	one hundred fifty-one billion
2,000,000,000,000	two trillion
8,000,000,000,000	eight trillion
214,000,000,000,000	two hundred fourteen trillion

Finally, we can mix these all together. For example:

1,854,000	one million, eight hundred fifty-four thousand
12,130,000	twelve million, one hundred thirty thousand
2,064,600,000	two billion, sixty-four million, six hundred thousand
8,284,347,418,671	eight trillion, two-hundred eight-four billion, three hundred forty-seven million, four hundred eighteen thousand, six hundred seventy one

With the numbers in this last set, especially the very last one, you can already seen one of the advantages mathematics has as a tool. It is much easier to write the number with numerical digits than it is to write it out in English.

Exercises

A) Write the following numbers numerically.

1) one million ____________________
2) two billion ____________________
3) five trillion ____________________
4) twenty-three million ____________________
5) fifty-seven trillion ____________________
6) two hundred thirty-eight billion ____________________
7) seventy-eight billion, three hundred eighty-eight million ____________________
8) four hundred fifty-three billion, eight hundred sixty-one million ____________________
9) five hundred sixty-nine trillion, four hundred seventy-one billion, two hundred thirty-two million ____________________
10) nine hundred fourth-seven trillion, six hundred ninety-two billion, twenty-one million, six hundred one ____________________

B) Write the names of the following numbers in English.

1) 3,000,000 ____________________
2) 5,000,000,000 ____________________
3) 7,000,000,000,000 ____________________
4) 56,000,000 ____________________
5) 73,000,000,000,000 ____________________
6) 868,000,000,000 ____________________
7) 97,142,000,000 ____________________
____
8) 885,479,000,000 ____________________
____
9) 421,933,593,000,000 ____________________
____
____

10) 237,807,046,612,739 ______________________________

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Sample Exam Questions

1) Which of the following is the numerical equivalent of five million?
   a) 5,000,000
   b) 50,000
   c) 500,000
   d) 5,000,000,000,000

2) Which of the following is the numerical equivalent of eight trillion?
   a) 8,000,000,000,000,000,000,000
   b) 8,000,000
   c) 8,000,000,000,000
   d) 800,000,000

3) Which of the following is the is the name for 500,000,000?
   a) five hundred thousand million
   b) five hundred trillion
   c) five hundred billion
   d) five hundred million

4) Which of the following is the name for 434,346,551,000?
   a) four hundred thirty-four trillion, three hundred forty-six billion, five hundred fifty-one thousand
   b) four hundred thirty-four billion, three hundred forty-six million, five hundred fifty-one thousand
   c) four hundred thirty four k’zillion, three hundred forty-six b’zillion, five hundred fifty-one million billion
   d) four hundred thirty-four million, three hundred forty-six thousand, five hundred fifty-one

5) Of the following four numbers, which is largest?
   a) 900,342,797
   b) one hundred three trillion
   c) 141,075
   d) two hundred ninety-two billion

Metric Prefixes & Abbreviations

While the naming convention for numbers is straightforward, writing the names out in English is rather cumbersome. Further, million, billion, and trillion sound very similar and can be confused when spoken. Finally, these are English words. People who don't speak English will not understand them. For all of these reasons when the Metric System was formally defined a set of prefixes for units and abbreviations for these prefixes were also defined. In 1991 the 19th General Conference on Weights and Measures extended the list of prefixes so that it encompasses numbers both small and large enough that they span all of the uses of modern science and technology. Some prefixes, kilo- (K), mega- (M), and giga- (G) have entered everyday language and are examples of ones we will most often encounter in astronomy. The complete set is:

Prefix	***Abbreviation***	***Number Name***	***Number in Scientific Notation***[2]
yotta-	Y	1 septillion	10^{24}
zetta-	Z	1 sextillion	10^{21}
exa-	E	1 quintillion	10^{18}
peta-	P	1 quadrillion	10^{15}
tera-	T	1 trillion	10^{12}
giga-	G	1 billion	10^{9}
mega-	M	1 million	10^{6}
kilo-	K or k	1 thousand	10^{3}
hecto-	h	1 hundred	10^{2}
deca-	da	ten	10^{1}
deci-	d	1 tenth	10^{-1}
centi-	c	1 hundredth	10^{-2}
milli-	m	1 thousandth	10^{-3}
micro-	µ[3]	1 millionth	10^{-6}
nano-	n	1 billionth	10^{-9}
pico-	p	1 trillionth	10^{-12}
femto-	f	1 quadrillionth	10^{-15}
atto-	a	1 quintillionth	10^{-18}
zepto-	z	1 sextillionth	10^{-21}
yocto-	y	1 septillionth	10^{-24}

The prefixes highlighted with a light gray background are the ones you will encounter regularly in introductory astronomy.

Metric prefixes are not used in place of numbers themselves but rather with units of measure. For example, you would not replace "billion" in a sentence with "giga", but you can write "one billion years" as one giga-year or more simply and compactly, 1 Gyr.

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2 The following section explains scientific notation.

3 This is the Greek letter mu, the Greek lowercase "m". It is the only non-roman letter used as a metric prefix. Greek letters and why we use them in astronomy are detailed in a later section.

Exercises

A) Write the following quantities with their metric prefixes. Use the following abbreviations for units:

seconds	s
years	yr
meters	m

1) 1 million seconds ______________________________

2) 3 billion years ______________________________

3) 700 billionths of a meter ______________________________

4) 1000 meters ______________________________

5) 1,000,000 kilometers (in units of meters) ______________________________

B) Write out the following abbreviated measures in English:

1) 8 Ms ______________________________

2) 4.5 Gyr ______________________________

3) 300 nm ______________________________

4) 8 μm ______________________________

5) 1 cm ______________________________

Sample Exam Questions

1) One billion, 500 million seconds is abbreviated _____.
   a) 1.5 Ms
   b) 15 Ms
   c) 15 Gs
   d) 1.5 Gs

2) 4 billion 650 million years is abbreviated _____.
   a) 4.65 gyr
   b) 4.65 Gyr
   c) 4.65 cyr
   d) 4 Gyr, 6.5 cMyr

3) How is 550 nm written out in English?
   a) Five hundred fifty million meters.
   b) Fifty-five hundred billionths of a meter
   c) Five hundred fifty billionths of a meter
   d) Five hundred fifty trillionths of a meter

4) How is 2 Gm written out in English?
   a) 2 billion meters
   b) 2 billionths of a meter
   c) 2 trillion meters
   d) 2 million meters

5) Which of the following is the longest amount of time?
   a) 100 million years
   b) 85 Myr
   c) 14 Gyr
   d) 3.78 billion years

Scientific Notation

While metric prefixes provide a way to write large quantities in an abbreviated way, they are not well suited to do calculations with. We need a short hand for writing large numbers that allows for mathematical manipulation. Scientific notation, sometimes called exponential notation, is this short hand.

The idea behind scientific notation is that when we have a very large or a very small number that has been determined by a measurement, we usually don't know all of the digits. For example, the distance to the Andromeda galaxy is:

$d = 2{,}900{,}000$ ly

This distance requires a large number to express, but notice we don't know the distance to the last light year. The only reason to write all of the digits in the number of light years is so we can record the size of the number. The zeros are place holders and in this case only the left-most two digits are actually known. Scientific notation preserves the known digits as a coefficient (in this case 2.9) and then multiplies this by 10 raised to an exponent to account for the place holding zeros. The exponent (sometimes called the "power of ten" or simply "power") is the same as *multiplying* 10 by itself that many times. For example:

$$\begin{aligned} d &= 2{,}900{,}000 \text{ ly} \\ &= 2.9 \times 1{,}000{,}000 \text{ ly} \\ &= 2.9 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \text{ ly} \\ &= 2.9 \times 10^6 \text{ ly.} \end{aligned}$$

The coefficient, the part we actually know from measurement is "2.9" and the exponent, the number of place holders to communicate the size of the number is "6" in 10^6. Another way to say this is we have to shift the decimal point 6 places to the *left* to convert the number to the coefficient.

Now the expression 2.9×10^6 is not that much of a short hand for 2,900,000. The first expression requires 7 characters to write and the second 9, not much of a difference. But suppose instead of light years we wrote the distance in meters, the standard unit of distance in the metric system. In this case the distance to the Andromeda galaxy is:

$$d = 27{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \text{ m} = 2.7 \times 10^{22} \text{ m.}$$

Now it is totally obvious why scientific notation is a good short hand. Imagine trying to type 22 zeros into a calculator and not loosing track as you go.

The same is done for very small numbers. The mass of the electron is:

$m_e = 9.10956 \times 10^{-31}$ kg (kilograms).

Written without scientific notation this is a very cumbersome number:

m_e = 0.000 000 000 000 000 000 000 000 000 000 910 956 kg

and again the advantage to writing it in scientific notation is immediately obvious. In this case the coefficient is 9.10956 (we know this mass more precisely, to more digits, than we know the distance to the Andromeda galaxy) and the exponent is -31. A negative exponent is the same as *dividing* by 10 this many times. Another way to say this is we have to shift the decimal point 31 places to the *right* to convert the number to the coefficient.

Basic arithmetic rules of manipulation for addition, subtraction, multiplication, and division apply to numbers written in scientific notation. In this class you won't have to do arithmetic by hand, so discussion will be saved for the next section, Scientific Notation on Your Calculator.

Exercises

A) Write the following numbers in scientific notation.

1) 800000 ____________________

2) 0.00000007 ____________________

3) 14000000000 ____________________

4) 0.0000008374 ____________________

5) 670000000000000000 ____________________

B) Write the following numbers in "normal" form.

1) 1.2×10^{2} ____________________

2) 3.78×10^{4} ____________________

3) 8.5×10^{-3} ____________________

4) 6.49×10^{-1} ____________________

5) 4×10^{0} ____________________

Sample Exam Questions

1) The average distance between the Earth and Sun is 93,000,000 miles. This can be written _____.
   a) 9.3×10^{7} miles
   b) 9.3×10^{-7} miles
   c) 9.3×10^{6} miles
   d) 9.3×10^{-6} miles

2) The wavelength of light the Sun radiates the most of is 0.000 000 55 m. This is equal to _____.
   a) 5.5×10^{7} m
   b) 5.5×10^{3} m
   c) 5.5×10^{-7} m
   d) 5.5×10^{-12} m

3) Light travels 5.9 trillion miles in one year. This can be written _____.
   a) 5.9×10^{-6} miles per year
   b) 5.9×10^{6} miles per year
   c) 5.9×10^{9} miles per year
   d) 5.9×10^{12} miles per year

4) Which of the following numbers is greatest?
   a) 4.7×10^{-12}
   b) 9.8×10^{9}
   c) 0.0×10^{12}
   d) 4.1×10^{0}

5) Which of the following numbers is smallest
   a) 4.7×10^{-12}
   b) 9.8×10^{9}
   c) 0.0×10^{12}
   d) 4.1×10^{0}

Scientific Notation on Your Calculator

Entering Numbers in Scientific Notation into Your Calculator

While the rules for writing and using scientific notation are standardized, calculator manufactures have not come up with a standard way to either enter or display numbers in scientific notation. If your calculator was manufactured in the last five to ten years it will *probably* follow one of the following conventions. If you think it doesn't check with your instructor and see if together you can figure out how your calculator works. But try the following first.

To enter a number in scientific notation you will use a button labeled either **EXP**, **EE**, or perhaps **EEX**. Locate this button on your calculator. Keep in mind it may be a "second function" button, that is a button usually labeled in a second color and which requires you to push the second function key to use.

To enter a number in scientific notation ***do not*** use any of the buttons labeled, **10^x**, **e^x** , **x^y** , **y^x** , or the keys labeled **× 1 0 ^**. ***If you use any of these you will likely get the wrong answer!***

For example, suppose you need to enter the number,

5.87×10^{13}

into your calculator. To do so you press **5 . 8 7 <EE> 1 3.** (The < and > simply indicate that the button is labeled with the letters in between. Do not look for a < and > button.) If your calculator does not have a button labeled **EE** then you likely enter this number by pressing, **5 . 8 7 <EXP> 1 3**. Again ***do not press*** the buttons **<10^x>**, **<e^x>**, **<x^y>**, **<y^x>**, or the keys **<×> <1> <0> <^>**. ***If you use any of these you will likely get the wrong answer!***

Once you have entered this number, or if this number is the result of a calculation, your calculator may display it in a number of possible ways. Unfortunately very few calculators will include the " × 10" part of the number but rather will imply it via one of the following conventions. Some possibilities are:

5.8E13
5.8 13 (a space separating the 8 and the 1)
5.8 ^{13}

Be sure you understand how your calculator displays scientific notation so you don't incorrectly write down what it is displaying. If for example the above number was the answer to a question, and on your answer sheet you write "5.8 ^{13}", you will get the question marked wrong. Mathematically "5.8 ^{13}" means to raise 5.8 to the 13^{th} power, not the same thing as 5.8×10^{13} power. ***It is your responsibility to translate correctly what your particular calculator displays.***

If instead of a large number like 5.87×10^{13} the number you are entering is less than 1, say 5.87×10^{-13}, you enter it in exactly the same way but you also have to enter the negative sign that goes along with the exponent. On most calculators ***the negative sign is not the same as the minus sign***. This may be somewhat counter-intuitive until you realize that to the calculator the minus sign means to subtract two numbers while the negative sign means the negative of a number. The minus sign, meaning subtraction, will be indicated by a **<–>** button, usually next to, below, or above the addition or plus button, **<+>**. The negative button will appear elsewhere and may be marked as **(–) , CHS** (for "change sign"), or **+/–** . To enter the number 5.87×10^{-13} press **5 . 8 7 <EE> <(–)> 1 3**.

Finally, if the number is a negative number with a negative exponent, then you will need to use the negative key twice. For example, to enter -5.87×10^{-13} into your calculator press **<(–)> 5 . 8 7 <EE> <(–)> 1 3.**

Step by step example:

Suppose you are doing an in class activity and your calculator displays the following number as an answer to a problem,

9800000000.

How do you write this number on the activity sheet? **Any number larger than 100,000 or smaller than 0.001 should be written in scientific notation** for simplicity and to prevent mistakes in transcription (writing too many or too few zeros). To write this number in scientific notation first determine the coefficient, the known part. In this case that is the 98 at the beginning, so write this as 9.8 (the coefficient should always have one digit to the left of the decimal and the rest to the right), then determine the exponent to put with the 10. To do this, simply count how many times you have to move the decimal to take it from where it is in the number to where it is in the coefficient. In this case 9 places to the *left*.[4] Then put these together in the scientific notation form:

9.8×10^{9}.

If the result from your calculator is a number less than 1, say:

0.00000786,

follow the same procedure except this time you will be shifting the decimal point to the

4 Most scientific calculators have some sort of menu/mode button that will set the calculator into a mode so that it will display *all* numbers in scientific notation. If you set your calculator this way you will not have to count zeros, but you will have to deal with small numbers being displayed in scientific notation. Every make and model of calculator does this differently, so if you want to use this functionality of your calculator, you will have to take the initiative and read your calculator's manual or otherwise figure out how to set it on your calculator.

right. In this case the coefficient is 7.86 and the exponent is -6. So in scientific notation the number is written as:

7.86×10^{-6}.

Often the number displayed on the calculator does not have place holding zero-digits but rather is just a long string of numbers. What do you do then? You have to consider where the answer came from, and how well you knew the measurements that went into calculating the number displayed. The number of known digits in the resulting number is the same as the number of known digits is the *least* well known of the numbers that went into the calculation. Suppose you calculated the following and got this result:

You should not write 2636231974 as the answer. If you do, you are stating that you know the answer to 10 digits, but you don't. The top two numbers going into the calculation were each known to 3 digits, and the bottom number was known to 6 digits. The answer is only known to the same number of digits as the least well known of the numbers going into the calculation, so in this case that is 3. The answer should be written in scientific notation and its coefficient should have 3 digits, 2.64, rounding up the last digit because the digit following it is greater than 5. The exponent is then the number of places the decimal point has to be moved going from the number to the coefficient, in this case that is 9 places to the left. So the number should be written as:

2.64×10^{9}.

Why not just write down what the calculator says? Because the calculator is mindless. It has no idea what you are doing. It can only do basic arithmetic. You have to be the mindful one and think a bit about what you are trying to work out. This may seem a bit cumbersome at first, but with practice it becomes easy and automatic, and saves you from writing out long strings of numbers.

Exercises

A) Type the following numbers into your calculator and hit the equal button, <=>.

1) 4.26×10^{15}

2) 5.31×10^{-13}

3) -8.742×10^{-18}

4) 57,874,000,000,000,000

5) 0.000 000 000 000 345

B) Do the following calculations on your calculator and write down the result in scientific notation. Be careful to write the correct number of digits in each coefficient.

1) = ____

2) = ____

3) = ____

4) = ____

5) = ____________________________

Sample Exam Questions

You will never be asked to demonstrate your skill with scientific notation or arithmetic manipulation on your calculator on an astronomy exam. However, you will need to know how to use these skills to do many other types of problems.

Percentage

Percentage is in very common usage in our society, especially in financial issues. Stores will often advertise a 50% off sale. Banks will list loan interest rates, and savings account interest rates as a percentage of the balance per year. Percentage is also often used in astronomy when referring to a fraction of something. For example, the Earth has had multi-cellular life for 16% of its existence, or approximately 50% of all solar systems have more than one star. To be a successful astronomy student or an informed consumer one must know what these percentages actually mean. Fortunately, gaining this skill is one of the more straightforward math skills to acquire.

"Percent" is an English word which comes from the Latin phrase meaning "per hundred" or "for every one hundred". So, 50% simply means, "50 out of every 100". Sixteen percent, 16%, means 16 per 100. If 16% of the class gets an A, then for every 100 students in the class 16 got an A.

The reason 100 is chosen is because for most things we are likely to use a percent for, 100 is large enough that we will likely get at least one. It is also very easy to use in our base-ten number system, because to change a decimal fraction into a percent, all we have to do is move the decimal place twice to the right (which of course is the same as multiplying by 100). Lets illustrate with some examples:

Example 1: You pick up an activity and discover you scored 8 out of the 10 possible points. You want to know what letter grade this is, but you only know how letter grades equate with percent scores. So what percent is 8 out of 10?

To calculate a percentage when you have 8 out of 10, or any number out of another, simply calculate the decimal fraction of the first number divided by the second:

8 out of 10 =

then move the decimal point twice to the right or if you prefer multiply by 100. Either will give you the same answer:

8 out of 10 = 0.800 = 80%.

What you are doing when you multiply by 100 is converting the decimal fraction to the "out of 100" part of percentage. So, in this case, you scored 80%. Now you can find out what letter grade you scored on this assignment.

Example 2: This is the same as example 1, but is a bit more scientific sounding, and on topic for an astronomy course. There are 128 know stars within 20 light years of the Earth. Of these 128 stars, 86 are M dwarf stars, the coolest type of main sequence star and the most abundant type of star.

What percentage of the stars within 20 ly of the Earth are M dwarf stars?

Again, to calculate the percentage, first calculate the decimal fraction of the number divided by the total. In this case:

86 out of 128 =

then move the decimal point twice to the right or if you prefer multiply by 100, and you get:

67.2% of the stars within 20 light years of the Earth are M dwarf stars.

Now lets take this a step further and do another common type of percentage calculation. Stars found nearby the Sun, some times called the solar neighborhood, are representative of all of the stars found in the disk of our galaxy. The disk of our galaxy contains approximately 100 billion stars, so based on the percentage we just calculated, how many M dwarfs are there in the disk of our galaxy?

The first thing to note is that stars found near the Sun are representative of all the stars in the disk of the galaxy. Since 67.2% of the stars near the Sun are M dwarfs, 67.2% of all the stars in the disk are also M dwarfs. So the question is asking, how many is 67.2% of 100 billion?

The nice thing about percentages is that to calculate the number if we know the percentage and the total number, all we have to do is multiply the percentage (written in its decimal fraction form) times the total number:

$$67.2\% \text{ of } 100 \text{ billion} = 0.672 \times 100 \text{ billion} = 0.672 \times (1.00 \times 10^{11}) = 6.72 \times 10^{10}.$$

So there are 6.72×10^{10} (67 billion, 200 million) M dwarf stars in the disk of our galaxy.

If this isn't obvious to you, we can work in out by comparison. Recall that 67.2% means 67.2 out of 100 and that this can be written as the fraction:

There are 100 billion stars in the disk of the galaxy, so to calculate the number of M dwarfs (represented by *x* in the following expression) by comparison we can write:

In English, "67.2 out of 100 is the same as what, *x*, out of 100 billion?" Cross multiplying we find:

and dividing each side by 100 we get:

So the number of M dwarfs in the disk of the galaxy is 6.72×10^{10}, the same number we calculated previously.

Exercises

A) For each of the following calculate the percentage.

1) 48 out of 100 = ____________________%

2) 6 out of 10 = ____________________%

3) 53 out of 67 = ____________________%

4) 75 out of 115 = ____________________%

5) 345 out of 1258 = ____________________%

B) For each of the following calculate the number.

1) 57% of 100 = ____________________

2) 70% of 10 = ____________________

3) 24% of 28 = ____________________

4) 14% of 221 = ____________________

5) 38% of 2.87×10^{11} = ____________________

Sample Exam Questions

You will never be asked to directly demonstrate your skill with percentage on an astronomy exam. However, you will need to know how to use these skills to do many other types of problems (like example 2 above), and will need them to understand your grades on exams and other assignments.

The Greek Alphabet

As we will see in the next section, **Understanding Equations,** we often use letters to represent certain quantities of measure such as distance, mass, etc. Some of these are obvious like *m* for mass, *d* for distance. This is a nice convention because it is easy to remember when looking back at a equation what *m* and *d* mean and both *m* and *d* are easier to write in an equation than *mass* or *distance*. There are also some standard mathematical conventions which use particular letters for certain purposes like, *x* and *y* as general coordinates, *i* as a counting variable, etc. With only 26 letters in the Roman alphabet, 52 if we count capitals, we rapidly run out of letters. An additional limitation is that some physical quantities start with the same letter of the alphabet, *distance* and *density* for example. If we used *d* for both, we could easily get confused.

To overcome this limitation we need more letters. We could use any symbols to augment the standard Roman alphabet. Traditionally many academics (including undergraduates!) spoke/read Latin and Greek. Latin uses the Roman alphabet, so Greek letters were the next most familiar in the academic environments and were used for many purposes. (Think of sororities and fraternities, for example.) So the Greek alphabet was chosen as the next set of symbols to represent many physical quantities.

The following is the upper and lower case Greek alphabet, in alphabetical order. The letters highlighted with a gray background are the ones you are most likely to encounter in introductory astronomy, and so you should learn their names and how to write them.

Α	α	alpha	Ν	ν	nu
Β	β	beta	Ξ	ξ	Xi
Γ	γ	gamma	Ο	ο	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ, ς	sigma[5]
Η	η	eta	Τ	τ	tau
Θ	θ	theta	Υ	υ	upsilon
Ι	ι	iota	Φ	φ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Some Greek letters are similar or identical to letters in the Roman alphabet. This is because the alphabet we use is in part derived from the Greek alphabet.

5 There are two forms of lowercase sigma. Written at the end of a word it is written ς. Written anywhere else it is written σ. In introductory astronomy we only use the Σ and σ forms.

Exercises

A) Practice printing each of the Greek letters highlighted in gray in the table. Practice until you can print each letter without looking at the table. This way you will be able to write each in your notes when you need to.

Alternatively, if you take notes on your computer, figure out how to enter them into your word processor. Some word processors will even let you set up keyboard macros for special characters, so you may want to define such macros for each of the Greek letters highlighted in gray in the table.

Sample Exam Questions

You will never be asked to draw or identify a Greek letter on an astronomy exam, however you will need to be able to interpret them in formulas and other situations.

Understanding Equations

Many students treat mathematical equations (or formulas) as mystical black boxes, memorized, used to “plug and chug”, but generally without meaning. Nothing could be further from the truth (or the best way to use them to get good grades!). Most equations in physical science convey a fundamental understanding about the physical world, the relationship between properties of an object or even properties of the entire Universe. Thinking about equations in this way is not only a good way to understand them, but will create understanding about the concepts they are representing. You should be able to “plug and chug” when you need to, but more fundamentally you should be able to read an equation and describe, in words, what it says about the physical world.

Example 1: We all understand the relationship between distance, velocity, and time. If you think you don’t, consider a road trip. It is about 120 miles from Bowling Green to Columbus, Ohio. It takes about two hours to drive from Bowling Green to Columbus. On average (including stops, turns, etc.) we drive about 60 miles per hour on a highway.

All of these statements are related. It takes two hours to drive to Columbus because of the distance Columbus is from Bowling Green and because of how fast we drive. If you drive faster than and average person, you can get there more quickly. If you drive slower, it takes more time. If instead of driving to Columbus you drive to a closer city, Toledo say, it will take you less time to get there. If you drive to a further city, Chicago for example, it takes more time to get there.

All of these ideas put together represent a “formula” relating distance, time, and driving velocity. We can describe it in words as in the above paragraph, or we can use the much more compact and precise language of mathematics and write it as the equation:

[illegible]

where *d* represents *distance*, *v* represents *velocity*, and *t* represents *time*.

With this equation you can derive all of the above relationships. For example, if the distance is fixed, d = 120 miles, then since *velocity* × *time* will always be equal to this number of miles it becomes clear that the larger the velocity, the smaller the time will be or the smaller the velocity, the larger the time will be. Consider, if average driving velocity is 60 miles per hour, the corresponding average trip time is 2 hours,

[illegible]

If you could drive faster than this average driving velocity, say 120 miles per hour, obviously you will get there faster. But how much less the time is, is exactly the same as how much faster you are driving since an increase in velocity will be offset by exactly the same *decrease* in time.

If you drive *twice* as fast you get there in *half* the time. If you drive *half* as fast (30 miles per hour) it takes *twice* as much time to get there:

When you look at a formula, relationships like these are the things you should be looking for.

Constants

Many formulas that you will encounter in your study of astronomy have one or more constants in them. A constant is a number which never changes. It is always the same in the equation. Usually the constant is in the equation so that the units of measure (kilograms, meters, seconds, etc.) balance. For example:

In this equation, λ represents the wavelength (color) of light, and T the surface temperature of a star. Since wavelength is a size, it is measured in meters, or billionths of a meter (nm) in the equation, and surface temperature is in *Kelvins*, the temperature scale used in astronomy. To make the equation an equality, we need the constant, 2,900,000 nm • K that tells us how the two units relate to each other in this relationship.

Since the constants are often cumbersome to write out, they are often represented by a letter just like the variables are. The equation that expresses the relationship between the physical size of a star, its temperature and how much light it emits is:

where L is the amount of light the star emits (luminosity), R is the radius of the star, and T is the surface temperature. The 4 and the π are numbers, and the σ is a constant that tells how square meters, Kelvins, and energy (amount of light) are related.

Keeping this is mind, if we look at this equation, we can tell some fundamental properties of stars. For example, if we have two stars with the same surface temperature (T has the same value) then the bigger star (the one with larger R) will produce the most light. If two stars are the same size and one is twice as hot as the other, the hotter star produces $2^4 = 16$ times as much light as the cooler star. If the stars produce the same amount of light and are the same size, they *must* have the same surface temperatures, and so on.

Exercises

A) Use the equation relating distance, time, and velocity ( ) to answer the following questions:

1) A drive from Bowling Green to the Northwest Suburbs of Chicago is about 300 miles. If you drive 60 miles per hour, how long does the drive take? (This is your chance to "plug and chug".)

2) If you drove half as fast as the speed in question 1, how long does the trip take? (Don't plug and chug. Look at the equation and think about it.)

3) If you drove twice as fast as the speed in question 1, how long does the trip take?

4) If instead of Chicago, you drove twice as far, but at twice the speed of question 1, how long would the trip take?

5) In general, if you drive twice as far and want to get there in half the time, how much faster do you have to drive?

B) The relationship between the pressure (P), temperature (T), and density (ρ) of a gas is , where k is a constant.

1) If the density of a gas doesn't change, and the temperature increases, what will happen to the pressure?

2) If the pressure of a gas is increased by a factor of 4 (made 4 times larger) and the density of the gas doubles, what happens to the temperature?

3) If the pressure of a gas is decreased by a factor of 4 but the density doesn't change, what happens to the temperature?

4) If we measure the temperature of a gas, and determine its density from experiment, what can we tell about the pressure without having to make any additional measurements?

5) If both the density and temperature of a gas are increased by a factor of 10, what happens to the pressure?

Sample Exam Questions

Exam questions will be based on other astronomical equations. Before such questions are relevant, we need to first discuss the equations. If you can do the above 10 questions you understand how to read and work with equations.

Answers to Exercises

The following are the answers to the exercises at the end of each section. Only check the answers once you have finished the exercises. If you look beforehand the exercises will do very little to help you learn the concepts.

Exercises – Names of Numbers

A) Write the following numbers numerically.

1)	one million	1,000,000
2)	two billion	2,000,000,000
3)	five trillion	5,000,000,000,000
4)	twenty-three million	23,000,000
5)	fifty-seven trillion	57,000,000,000,000
6)	two hundred thirty-eight billion	238,000,000,000
7)	seventy-eight billion, three hundred eighty-eight million	78,388,000,000
8)	four hundred fifty-three billion, eight hundred sixty-one million	453,861,000,000
9)	five hundred sixty-nine trillion, four hundred seventy-one billion, two hundred thirty-two million	569,471,232,000,000
10)	nine hundred fourth-seven trillion, six hundred ninety-two billion, twenty-one million, six hundred one	947,692,021,000,601

B) Write the names of the following numbers in English.

1)	3,000,000	three million
2)	5,000,000,000	five billion
3)	7,000,000,000,000	seven trillion
4)	56,000,000	fifty-six million
5)	73,000,000,000,000	seventy-three trillion
6)	868,000,000,000	eight hundred sixty-eight billion
7)	97,142,000,000	ninety-seven billion, one hundred forty-two million

8)	885,479,000,000	eight hundred eighty-five billion, four hundred seventy-nine million
9)	421,933,593,000,000	four hundred twenty-one trillion, nine hundred thirty-three billion, five hundred ninety-three million
10)	237,807,046,612,739	two hundred thirty-seven trillion, eight hundred seven billion, forty-six million, six hundred twelve thousand, seven hundred thirty nine

Sample Exam Questions – Names of Numbers

1) Which of the following is the numerical equivalent of five million?
   a) 5,000,000
   b) 50,000
   c) 500,000
   d) 5,000,000,000,000

2) Which of the following is the numerical equivalent of eight trillion?
   a) 8,000,000,000,000,000,000,000
   b) 8,000,000
   c) 8,000,000,000,000
   d) 800,000,000

3) Which of the following is the is the name for 500,000,000?
   a) five hundred thousand million
   b) five hundred trillion
   c) five hundred billion
   d) five hundred million

4) Which of the following is the name for 434,346,551,000?
   a) four hundred thirty-four trillion, three hundred forty-six billion, five hundred fifty-one thousand
   b) four hundred thirty-four billion, three hundred forty-six million, five hundred fifty-one thousand
   c) four hundred thirty four k’zillion, three hundred forty-six b’zillion, five hundred fifty-one million billlion
   d) four hundred thirty-four million, three hundred forty-six thousand, five hundred fifty-one

5) Of the following four numbers, which is largest?
   a) 900,342,797
   b) one hundred three trillion
   c) 141,075
   d) two hundred ninety-two billion

Exercises – Metric Prefixes and Abbreviations

A) Write the following quantities with their metric prefixes. Use the following abbreviations for units:

seconds	s
years	yr
meters	m

1) 1 million seconds *1 Ms*

2) 3 billion years *3 Gyr*

3) 700 billionths of a meter *700 nm*

4) 1000 meters *1 km*

5) 1,000,000 kilometers (in units of meters) *1 Gm*

B) Write out the following abbreviated measures in English:

1) 8 Ms *8 million seconds*

2) 4.5 Gyr *four and a half billion years or*
four billion, five hundred million years

3) 300 nm *three hundred billionths of a meter*

4) 8 µm *eight millionths of a meter*

5) 1 cm *one hundredth of a meter*

Sample Exam Questions

1) One billion, 500 million seconds is abbreviated _____.
a) 1.5 Ms
b) 15 Ms
c) 15 Gs
d) 1.5 Gs

2) 4 billion 650 million years is abbreviated _____.
a) 4.65 gyr
b) 4.65 Gyr
c) 4.65 cyr
d) 4 Gyr, 6.5 cMyr

3) How is 550 nm written out in English?
   a) Five hundred fifty million meters.
   b) Fifty-five hundred billionths of a meter
   c) Five hundred fifty billionths of a meter
   d) Five hundred fifty trillionths of a meter

4) How is 2 Gm written out in English?
   a) 2 billion meters
   b) 2 billionths of a meter
   c) 2 trillion meters
   d) 2 million meters

5) Which of the following is the longest amount of time?
   a) 100 million years
   b) 85 Myr
   c) 14 Gyr
   d) 3.78 billion years

Exercises – Scientific Notation

A) Write the following numbers in scientific notation.

1) 800000 *8×10^5*

2) 0.00000007 *7×10^{-8}*

3) 14000000000 *1.4×10^{10}*

4) 0.0000008374 *8.374×10^{-7}*

5) 670000000000000000 *6.7×10^{17}*

B) Write the following numbers in "normal" form.

1) 1.2×10^2 *120*

2) 3.78×10^4 *37,800*

3) 8.5×10^{-3} *0.0085*

4) 6.49×10^{-1} *0.649*

5) 4×10^0 *4*

Sample Exam Questions

1) The average distance between the Earth and Sun is 93,000,000 miles. This can be written _____.
   a) 9.3×10^7 miles
   b) 9.3×10^{-7} miles
   c) 9.3×10^6 miles
   d) 9.3×10^{-6} miles

2) The wavelength of light the Sun radiates the most of is 0.000 000 55 m. This is equal to _____.
   a) 5.5×10^7 m
   b) 5.5×10^3 m
   c) 5.5×10^{-7} m
   d) 5.5×10^{-12} m

3) Light travels 5.9 trillion miles in one year. This can be written _____.
   a) 5.9×10^{-6} miles per year
   b) 5.9×10^6 miles per year
   c) 5.9×10^9 miles per year
   d) 5.9×10^{12} miles per year

4) Which of the following numbers is greatest?

a) 4.7×10^{-12}
b) 9.8×10^{9}
c) 0.0×10^{12}
d) 4.1×10^{0}

5) Which of the following numbers is smallest

a) 4.7×10^{-12}
b) 9.8×10^{9}
c) 0.0×10^{12}
d) 4.1×10^{0}

Exercises – Scientific Notation on Your Calculator

A) Type the following numbers into your calculator and hit the equal button, <=>.

1) 4.26×10^{15}

2) 5.31×10^{-13}

3) -8.742×10^{-18}

4) 57,874,000,000,000,000

5) 0.000 000 000 000 345

No answers to display. Your "answer" is correctly getting the number typed into your calculator.

B) Do the following calculations on your calculator and write down the result in scientific notation. Be careful to write the correct number of digits in each coefficient.

1) = *4.5×10^{10}*

2) = *5.71×10^{16}*

3) = *$2.25 \times 10^{1} = 22.5$*

4) = *8.72×10^{7}*

5) = *3.54×10^{22}*

Sample Exam Questions

You will never be asked to demonstrate your skill with scientific notation or arithmetic manipulation on your calculator on an astronomy exam. However, you will need to know how to use these skills to do many other types of problems.

Exercises – Percentage

A) For each of the following calculate the percentage.

1) 48 out of 100 = *48%*

2) 6 out of 10 = *60%*

3) 53 out of 67 = *79%*

4) 75 out of 115 = *65%*

5) 345 out of 1258 = *27.4%*

B) For each of the following calculate the number.

1) 57% of 100 = *57*

2) 70% of 10 = *7*

3) 24% of 28 = *6.7*

4) 14% of 221 = *31*

5) 38% of 2.87×10^{11} = 1.1×10^{11}

Sample Exam Questions

You will never be asked to demonstrate your skill with percentage on an astronomy exam. However, you will need to know how to use these skills to do many other types of problems, and will need them to understand your grades on exams and other assignments.

Exercises

A) Use the equation relating distance, time, and velocity ( ) to answer the following questions:

1) A drive from Bowling Green to the Northwest Suburbs of Chicago is about 300 miles. If you drive 60 miles per hour, how long does the drive take? (This is your chance to "plug and chug".)

If you drive 60 miles per hour the drive takes:

.

The drive takes 5 hours.

2) If you drove half as fast as the speed in question 1, how long does the trip take? (Don't plug and chug. Look at the equation and think about it.)

If you drive have as fast, v is half as much, so to cover the same distance t has to be twice as much. If you drive half as fast, the trip takes twice as long.

3) If you drove twice as fast as the speed in question 1, how long does the trip take?

By similar reasoning to that of question 2, if you drive twice as fast the trip takes half as long.

4) If instead of Chicago, you drove twice as far, but at twice the speed of question 1, how long would the trip take?

In this case d is twice as much, but so is v, so the time is unchanged. This trip takes 5 hours just like the one in question one did.

5) In general, if you drive twice as far and want to get there in half the time, how much faster do you have to drive?

If you drive twice as far, d is twice as large, and want to get there in half the time, t is half the value, then you need to drive 4 times as fast.

B) The relationship between the pressure (P), temperature (T), and density (ρ) of a gas is [illegible], where k is a constant.

1) If the density of a gas doesn't change, and the temperature increases, what will happen to the pressure?

If we don't change ρ but increase T, then P will increase the same amount as T.

2) If the pressure of a gas is increased by a factor of 4 (made 4 times larger) and the density of the gas doubles, what happens to the temperature?

The temperature would also have to double so when combined with the doubling of the density, we get 4 times as much pressure.

3) If the pressure of a gas is decreased by a factor of 4 but the density doesn't change, what happens to the temperature?

If the pressures is one quarter as much and density is unchanged, the temperature would also drop to one quarter its value.

4) If we measure the temperature of a gas, and determine its density from experiment, what can we tell about the pressure without having to make any additional measurements?

We can tell exactly what the pressure is just from our knowing the temperature and density. These physical properties always follow the [illegible] *relationship, so we can use it to determine the pressure.*

5) If both the density and temperature of a gas are increased by a factor of 10, what happens to the pressure?

The pressure would increase by a factor of 100, 10 × 10.

Sample Exam Questions

Exam questions will be based on other astronomical equations. Before such questions are relevant, we need to first discuss the equations. If you can do the above 10 questions you understand how to read and work with equations.